Column collapse of granular rods

M. Trepanier and Scott V. Franklin*

Department of Physics, Rochester Institute of Technology

(Dated: June 23, 2010)

We investigate the collapse of granular rodpiles as a function of particle (length/diameter) and pile (height/radius) aspect ratio. We find that, for all particle aspect ratios below 24, there exists a critical height H_l below which the pile never collapses, maintaining its initial shape as a solid, and a second height H_u above which the pile always collapses. Intermediate heights between H_l and H_u collapse with a probability that increases linearly with increasing height. The linear increase in probability is independent of particle length, width, or aspect ratio. When piles collapse, the runoff scales as a piecewise power-law with pile height, with $r_f \sim \tilde{H}^{1.2\pm0.1}$ for pile heights below $\tilde{H}_c \approx 0.74$ and $r_f \approx H^{0.6\pm0.1}$ for taller piles.

PACS numbers: 45.70.Ht

I. INTRODUCTION

The collapse and flow of granular materials is fundamental to catastrophic events such as avalanches, landslides, and pyroclastic flows. Understanding how initially stationary piles of granular material collapse, how far the material travels before stopping, and the final pile geometry, is an important step in efforts to deal with the practical consequences of such natural phenomena.

Previous experimental investigations in granular column collapse [1, 2] have focused on cylindrical piles (initial radius r_0) of varying height H. The radius of the collapsed pile, r_f, scaled as $\tilde{r} \equiv (r_f - r_0)/r_0$, shows a power-law dependence on dimensionless height $\tilde{H} \equiv H/r_0$. There is a crossover in the power-law exponent when $\tilde{H}$ becomes larger than a critical height $\tilde{H}_c$, although there is not consensus on the specific value of $\tilde{H}_c$. Lube et al. [1, 2] found

$$\tilde{r} = \begin{cases} \tilde{H}^{1.} & \tilde{H} < 1.7 \\ \tilde{H}^{0.5} & \tilde{H} > 1.7 \end{cases} \qquad (1)$$

for a variety of irregular granular materials, independent of surface roughness. The crossover was also observed to coincide with a slight change in the final pile shape.

Lajeunesse et al. [3, 4] studied more regular materials (smooth glass beads) and also observed a change in scaling of the runoff, but found the critical height to be $\tilde{H}_c \approx 0.74$. The collapse of low ($\tilde{H} < 0.74$) piles resulted in a mesa-like final state, with some fraction of the top surface remaining horizontal. The horizontal plateau shrank with increasing $\tilde{H}$, finally disappearing for $\tilde{H} > 0.74$. Both Lube and Lajeunesse found little change in behavior for a variety of particle types or surface roughness. Their studies encompassed fine and coarse quartz sand, salt, couscous, rice, sugar, and smooth glass beads.

It is well known that rod-like granular materials of sufficient aspect ratio $\alpha \equiv L/d$ can support 90° angles of repose, and thus piles of these granular materials maintain the shape of the container in which they were initially formed (c.f. figure 1 (right)). Philipse [5–8] first studied this solid plug-like behavior, and observed that particles with aspect ratio greater than ≈ 35 could not be poured from a container, but rather emerged as a solid, a behavior has been assumed to be due to geometric entanglement of the particles.
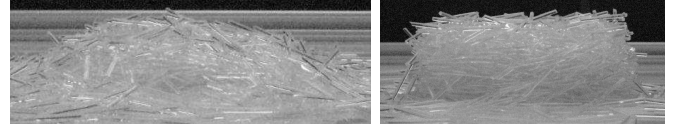

FIG. 1: Rod-like piles of moderate height may collapse (left), but may also retain their original, cylindrical shape (right). Shown are results from two experiments on aspect ratio $\alpha = 16$ particles with identical initial heights. We find the probability of collapse increases linearly with pile height.

This has led to a great deal of research on rods of various aspect ratio. Villarruel et al. [9] found that rods of aspect ratio ≈ 4 constrained to a cylinder would, when tapped, align with the cylinder walls and compact. Lumay and Vandewalle [10] extended this experiment to much larger aspect ratios, and developed a 2d lattice model that reproduced the salient features of the compaction curves. Lumay and Vandewalle subsequently [11] used a two-dimensional experiment to highlight the importance of rotation in rod rearrangements. Blouwolff and Fraden [12] investigated the contact number in 3d packings, and Stokely et al. [13] looked at static properties of 2d packings. Pournin et al. [14, 15] have written discrete element method (DEM) simulations of spherocylindrical particles that show crystallization and ordering under various excitations. Recently, Hidalgo et al. [16] looked at the role particle shape plays on stress propagation through a granular packing, while Azema and Radji[17] looked at stress-strain relations of rod packings under shear. Finally, there has been some attention

paid to the nature of random packings. Wouterse et al. [18] looked at the microstructure of random packings of spherocylinders and Zeravcic et al. [19] have looked at the excitations of random ellipsoid packings at the onsite of jamming. Despite all this attention, there have been no structural characteristics identified that explain the increased rigidity of large aspect-ratio particles.

Desmond et al. [20] studied the transition from classically granular to solid-like behavior by measuring the force on a small object pulled through a rodpile. They found characteristically granular slip-stick behavior for low aspect ratio particles, while large aspect ratio particles responded as a solid, moving as a single coherent unit. Desmond et al. were the first to observe the existence of a transition region separating the granular and solid behaviors. Moderate aspect-ratio particles displayed both granular and solid behavior, often alternating between the two. Typically a pile would briefly display solid-like behavior and move coherently, before collapsing around the intruder. Desmond et al. mapped the behavior in a two-dimensional parameter space of particle aspect ratio and inverse container size, but there was no quantitative measure of the transition region.

II. EXPERIMENTAL SETUP

Our experimental apparatus follows that of Lube and Lajeunesse [1–4]. An acrylic cylinder with inner radius r_0 is filled with material to height H and then quickly lifted vertically. Two cylinder radii were tested, $r_0 = 5.4 \pm 0.1$ cm and 7.3 ± 0.1 cm. We observed no dependence on cylinder radius and, unless otherwise noted, all reported results are from the smaller cylinder. The cylinder was filled by dropping particles in from the top. Two orthogonal cross-beams spanned the top entrance to the cylinder and randomized the particles' orientation as they fell into the cylinder. The cross-beams also broke up any clumps that existed prior to pouring.

The cylinder was raised with speeds ranging from 1-10 cm/s; little dependence on cylinder speed was observed. After the pile had collapsed, the distance from the cylinder's central axis to the pile edge, defined as the midpoint of the farthest particle in contact with the main pile, was measured in four directions ($\pm x, \pm y$) and averaged.

Rods were cut from acrylic and Teflon rod stock. Particle diameters were $D = 0.16 \pm 0.01 - 0.95 \pm$ cm, lengths $L = 2.5 \pm 0.2 - 7.6 \pm 0.2$ cm, resulting in a range of aspect ratio from $2.6 - 47.5$. The nominal coefficient of friction acrylic is $\mu_{acr} = 0.8$; that of Teflon $\mu_T = 0.04$. Ordinary sand was tested as a control, and compared with the results of Lube and Lajeunesse et al.

III. SCALING OF RUNOFF WITH HEIGHT

Following Lube, we define the runoff as the change in pile radius, normalized by the initial radius: $\tilde{r} \equiv (r_f -$

$r_0)/r_0$. As shown in Fig. 2, sand, acrylic rods of aspect ratio 8, 10, and 16, and Teflon rods of aspect ratio 8 and 16 all show a power-law dependence on normalized pile height $\tilde{H} \equiv H/r_0$ with change in exponent at a critical height $\tilde{H}_c$. To determine the the exponents and crossover height we perform a 3-parameter least-squares fit of the data to the piece-wise power-law function

$$\tilde{r} = \begin{cases} \tilde{H}^{\alpha_1} & \tilde{H} < \tilde{H}_c \\ \tilde{H}^{\alpha_2} & \tilde{H} > \tilde{H}_c \end{cases} \tag{2}$$

The minimization is done analytically for α_1 and α_2 and numerically for $\tilde{H}_c$. We have done this fitting both for every individual data run, averaging the resulting exponents and transition point, as well as for the aggregate collection of all data. Both analyses fall within the same standard-deviation uncertainty for the parameters, with $\alpha_1 = 1.2 \pm 0.1, \alpha_2 = 0.6 \pm 0.1$, and $\tilde{H}_c = 1.1 \pm 0.3$, or, putting these values into Eq. 2

$$\tilde{r} = \begin{cases} \tilde{H}^{1.2 \pm 0.1} & \tilde{H} < 1.1 \pm 0.3 \\ \tilde{H}^{0.6 \pm 0.1} & \tilde{H} > 1.1 \pm 0.3 \end{cases} \tag{3}$$
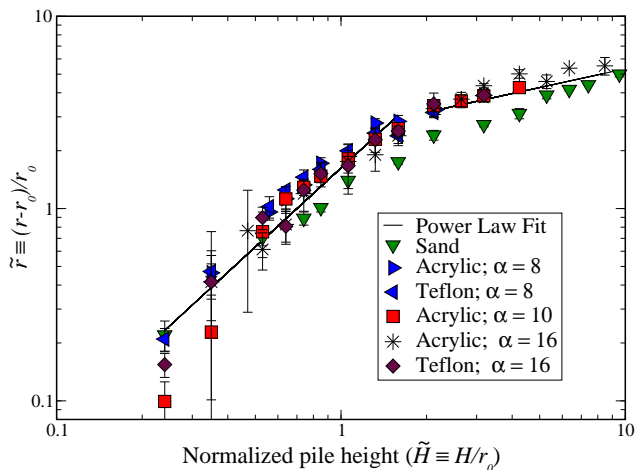

FIG. 2: The runoff distance $\tilde{r}$ of a collapsed granular pile shows a power-law dependence on initial height $\tilde{H}$, with a transition between linear and square-root scaling. Both radius and pile height $\tilde{H}$ are made dimensionless by the initial pile radius r_0. For $\tilde{H} <\approx 1.1$ the runoff increases linearly with $\tilde{H}$, whereas for larger piles the runoff increases as $\tilde{H}^{1/2}$ (solid lines are power law fits). Both scaling behaviors, as well as the transition point, arise from conservation of volume arguments that assume all material outside an internal, stable cone collapses.

The range of heights tested is only about a decade for the lower region, and less than a decade for the upper region, and so we only assert that our results are consistent with the findings of Lube et al. [2] and Lajeunesse et al. [3, 4].

As worked out by Lajeunesse [3], all of the behavior in Fig. 2 — the two power-law scaling exponents and the

crossover between the two — can be explained by conservation of volume, assuming the material does not compact as it collapses. The crossover height occurs when the dimensionless pile height is such that

$$\tilde{H}_0 = \tan\theta_c,$$

where θ_c is the dynamic angle of repose. Our observed transition height of $\tilde{H}_0 = 1.1 \pm 0.3$ implies an angle of repose of about $\theta_c \approx 47° \pm 7°$, slightly higher than that found by Lajeunesse for round and irregular particles, indicating a slightly increased structural stability of rod-piles.

To conclude this section, the runoff of rods of even moderate aspect ratios follows similar scaling as that of sand. The comparable transition point indicates a similar angle of repose suggesting that, when the pile collapses, the cylindrical particles roll much like spheres. The square-root scaling of runoff at large pile heights indicates that the dynamics of pile collapse are also similar, with an immobile inner core setting the final pile height.

IV. SOLIDITY OF MODERATE ASPECT-RATIO PARTICLES

One of the more striking behaviors of large aspect-ratio granular material is their ability to maintain the shape of their initial container. As expected, piles formed from particles with aspect ratio (L/d) greater than 24 never collapsed, maintaining their cylindrical shape. While 24 is slightly lower than the value commonly assumed necessary to maintain plug-like behavior ([5, 6, 12, 20]), it is consistent with the finding of Desmond et al. that finite-size container effects can induce plug-like behavior at lower aspect ratios [20]. Surprisingly, however, we find that this solid-like behavior manifests in moderate aspect-ratio particles as well (see Fig. 1 (right)).

The behavior of piles formed from particles with intermediate aspect ratios $4 < \alpha < 24$ could not be predicted a priori, with experiments with visually equivalent initial conditions producing dramatically different results. Shown in Fig. 1 are two piles that resulted with particles of aspect ratio $\alpha = 16$ with relatively low initial pile heights. As can be seen, one pile collapsed, resulting in a mound formation with a relatively well-defined angle of repose, while the other did not, remaining cylindrical. When the pile collapsed, its runoff obeyed the scaling behavior shown in Fig. 2.

For all aspect ratios below 24, a critical height exists above which the pile always collapsed. Additionally, for each of these aspect ratios there exists a minimum height below which the pile never collapsed. Increasing the particle aspect ratio increased these critical heights, and so the initial radius does not seem the appropriate quantity by which to characterize this behavior. We find, however, that these critical heights are the same for all aspect ratios tested when scaled by the particle length L. This is shown in Fig. 3 for acrylic particles. The dashed lines

exist to guide the eye and reveal qualitative changes in behavior when the pile height is less than $L/4$, between $L/4$ and $3L/4$, and greater than $3L/4$. It should be noted that the data in Fig. 3 come from a variety of particle diameters and initial cylinder radii, and so argue that it is the particle length that determines the maximum pile height. Reducing particle friction narrowed, but did not eliminate, the transition region.
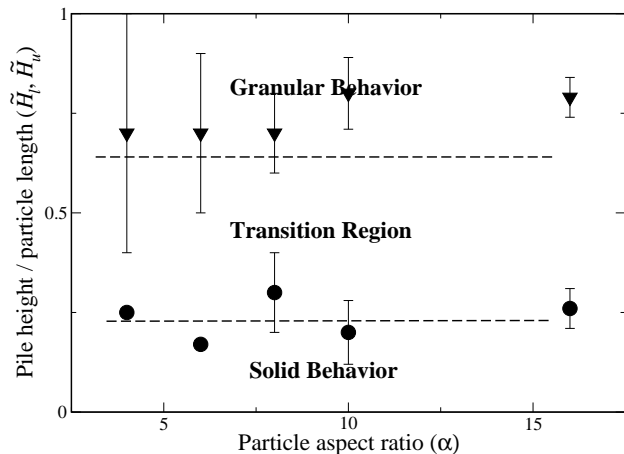

FIG. 3: The critical heights that determine whether a pile acts as a solid or granular material, scaled by particle length, are independent of aspect ratio. The solid and granular regions are separated by a transition region in which the pile has a probability of collapsing, although there are no visible differences between piles that collapse and those that do not.

The probability for a pile to collapse increases as one moves through the transition region. We use the upper and lower critical heights $\tilde{H}_l$ and $\tilde{H}_u$ to parametrize the height, defining an height order parameter that is 0 at $\tilde{H}_l$ and 1 at $\tilde{H}_u$:

$$\mathbf{H} \equiv \frac{\tilde{H} - \tilde{H}_l}{\tilde{H}_u - \tilde{H}_l}. \tag{4}$$

In Figure 4 we plot the collapse probability as a function of this order parameter $\mathbf{H}$ for all of our data, encompassing the complete range of aspect ratios that collapse (4-20) and two different particle diameters. Figure 4 shows that, when scaled as per Eq. 4, all the data collapse onto a straight line between (0,0) (the point at which no piles collapse) to (1,1) (where all piles collapse). The data in Fig. 4 are grouped by height into bins of scaled width 0.1, corresponding to the horizontal error bars. The vertical error bars represent one standard deviation of the average collapse probability of all data in that bin.

Figures 3 and 4 represent the first quantitative description of the transition region in which the material can behave either as a solid or as a granular material. This region is unique to large aspect-ratio materials, and appears to exist even when the aspect ratio is moderate (~ 4). Computational work is underway to determine
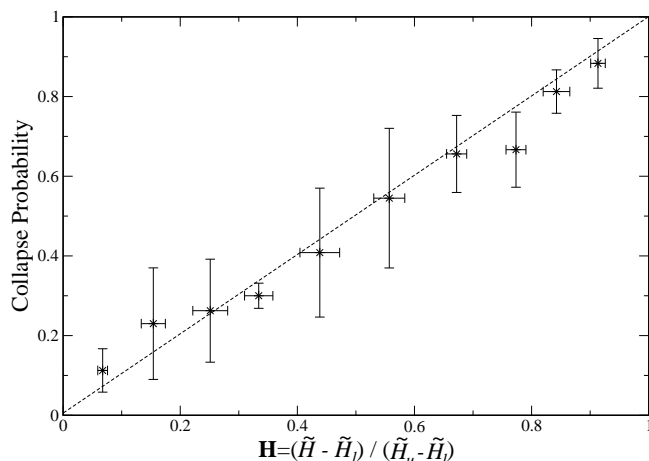
FIG. 4: The probability of collapse within the transition region increases linearly with pile height scaled by the critical heights. This behavior is independent of particle length, width, and aspect ratio. The normalization by critical heights is done so that the origin is the lower transition point and (1,1) is the higher transition point.

what internal characteristics (e.g. distribution of forces or percolating force chains) might be responsible for the different behavior in superficially similar piles.

The region of bistability was observed in two different cylinder radii, and the critical heights are normalized by the cylinder radius. Both cylinders were larger than a particle length; it is not clear how piles formed in extremely small containers (compared to the particle length) would respond. In this case particles could not orient horizontally, and this may reduce the stability of any pile formed. Desmond et al. [20] found that extremely large containers allowed for a more granular response to an intruder, but it is not clear that the same behavior exists in this geometry. We saw no decrease in pile rigidity in the larger cylinder tested.

V. CONCLUSIONS

We have investigated the collapse of granular columns consisting of cylindrical rods. Interestingly, when the pile collapses, it does so in a manner very similar to that of spheres or irregular sand. The runoff distance, when scaled by the initial pile radius, initially scales linearly with the normalized pile height. This corresponds to the partial collapse of the pile, involving only the material at the outer edges. The final pile shape is that of a truncated cone, with outer radius determined by the angle of repose of the material. At a critical pile height, also determined by the angle of repose, there is a crossover to a different scaling, where the runoff grows as the square-root of scaled height. This indicates an immobile inner cone of material, at the angle of repose, and that the final pile height is set by the initial radius. Both the scaling behavior, and the location of the cross-over height, are quantitatively consistent with earlier work on spheres, and so we conclude that, when rodpiles collapse, they are quantitatively the same as ordinary sand or spheres.

Large-aspect ratio particles do display a fundamentally different behavior from that of spheres, however, in their ability to maintain the shape of their initial container, essentially acting as a solid. We have shown that the onset of this behavior with increasing aspect ratio is not sudden, but manifests in even moderate aspect ratios under suitable experimental geometries. For a given aspect ratio there exist critical pile heights that demarcate the solid and granular behaviors, and the transition region separating the two. The transition region is characterized by a probability of solid/granular response, with a linear transition between the two. The transition heights, when scaled by particle length, are independent of particle aspect ratio. This behavior persists until the aspect ratio is quite large (> 24), at which all piles are essentially solid. Future research will investigate the internal characteristics that determine the pile response, as well as the behavior of the critical heights with increasing particle aspect ratio and pile radius.

[1] G. Lube, H. E. Huppert, R. S. J. Sparks, and A. Freundt, Physical Review E **72**, 041301 (2005).

[2] G. Lube, H. E. H. ad R. Stephen J. Sparks, and M. A. Hallworth, Journal of Fluid Mechanics **508**, 175 (2004).

[3] E. Lajeunesse, A. Mangeney-Castelnau, and J. P. Vilotte, Physics of Fluids **16**, 2371 (2004), URL http://link.aip.org/link/?PHF/16/2371/1.

[4] E. Lajeunesse, J. B. Monnier, and G. M. Homsy, Physics of Fluids **17**, 103302 (pages 15) (2005), URL http://link.aip.org/link/?PHF/17/103302/1.

[5] A. P. Philipse, Langmuir **12**, 1127 (1996).

[6] A. P. Philipse and A. Verberkmoes, Physica A **235**, 186 (1997).

[7] A. P. Philipse and S. G. J. M. Kluijtmans, Physica A **274**, 516 (1997).

[8] S. R. Williams and A. P. Philipse, Phys. Rev. E **67**,

051301 (2003).

[9] F. X. Villarruel, B. E. Lauderdale, D. M. Mueth, and H. M. Jaeger, Physical Review E **61**, 6914 (2000).

[10] G. Lumay and N. Vandewalle, Physical Review E **70**, 051314 (2004).

[11] G. Lumay and N. Vandewalle, Physical Review E **74**, 021301 (2006).

[12] J. Blouwolff and S. Fraden, Europhysics Letters **76**, 1095 (2006).

[13] K. Stokely, A. Diacou, and S. V. Franklin, Physical Review E **67**, 051302 (2003).

[14] L. Pournin, M. WEber, M. Tsukahara, J. A. Ferrez, M. Ramaioli, and T. M. Libling, Granular Matter **7**, 119 (2005).

[15] M. Ramaioli, L. Pournin, and T. M. Liebling, Physi-

cal Review E (Statistical, Nonlinear, and Soft Matter Physics) **76**, 021304 (2007).

[16] R. C. Hidalgo, I. Zuriguel, D. Maza, and I. Pagonabarraga, Phys. Rev. Lett. **103**, 118001 (2009).

[17] E. Azéma and F. Radjaï, Phys. Rev. E **81**, 051304 (2010).

[18] A. Wouterse, S. R. Williams, and A. P. Philipse, Journal of Physics: Condensed Matter **19**, 406215 (2007), URL http://stacks.iop.org/0953-8984/19/i=40/a=406215.

[19] Z. Zeravcic, N. Xu, A. J. Liu, S. R. Nagel, and W. van Saarloos, EPL (Europhysics Letters) **87**, 26001 (2009), URL http://stacks.iop.org/0295-5075/87/i=2/a=26001.

[20] K. Desmond and S. V. Franklin, Physical Review E **73**, 031306 (2006).